Phil 2310
Fall 2010

Assignment 5: This homework is due by the beginning of class on Fri, Oct 1st.

Part I. In each of the following cases, determine whether the sequent is valid by either giving an invalidating assignment, or by giving some argument that there is none.

1. $(P \rightarrow Q) \wedge (Q \rightarrow \neg P), R \rightarrow \neg P \vdash R \rightarrow (P \wedge Q)$
2. $(Q \rightarrow R) \rightarrow S, (U \vee R) \rightarrow Q \vdash (U \vee Q) \rightarrow S$
3. $(((Q \rightarrow R) \rightarrow R) \rightarrow P) \rightarrow P, P \rightarrow (Q \wedge \neg Q) \vdash Q \vee R$
4. $P \rightarrow \neg P, \neg R \rightarrow R \vdash P \wedge (\neg R \wedge S)$
5. $(P \rightarrow Q) \rightarrow R, (R \wedge S) \rightarrow U \vdash (\neg U \wedge \neg Q) \rightarrow \neg S$
6. $\neg(P \rightarrow Q), R \wedge (Q \vee S) \vdash (R \wedge U) \vee (P \wedge \neg U)$

Part II. Assume that it is possible to construct a proof in $\mathcal{F}_T$ from the premises P_1, P_2, P_3 to the conclusion Conc. Which of the following MUST be true? (The correct answer may be any number of these).

1) Conc is a logical consequence of $\{P_1, P_2, P_3\}$
2) $\neg$Conc is not a logical consequence of $\{P_1, P_2, P_3\}$
3) $\{P_1, P_2, P_3\}$ is a consistent set
4) $\{P_1, P_2, P_3\}$ is an inconsistent set
5) $\{P_1, P_2, P_3, \text{Conc}\}$ is an inconsistent set
6) $\{P_1, P_2, P_3, \neg\text{Conc}\}$ is an inconsistent set
7) $\{P_2, P_3, \neg\text{Conc}\}$ is an inconsistent set
8) $\{P_2, P_3, \neg\text{Conc}\}$ is a consistent set
9) $\{\neg P_1, P_2, P_3, \text{Conc}\}$ is an inconsistent set
10) $\{\neg P_1, \neg P_2, \neg P_3, \text{Conc}\}$ is a consistent set
11) $\neg P_1$ is a logical consequence of $\{P_2, P_3, \text{Conc}\}$
12) $\neg P_1$ is a logical consequence of $\{P_2, P_3, \neg\text{Conc}\}$
13) $\neg P_3$ is provable in $\mathcal{F}_T$ from $\{P_1, P_2, \neg\text{Conc}\}$
14) P_3 is provable in $\mathcal{F}_T$ from $\{P_1, P_2, \text{Conc}\}$
14) $P_1 \rightarrow$ Conc is provable in $\mathcal{F}_T$ from $\{P_2, P_3\}$
15) $P_1 \leftrightarrow$ Conc is provable in $\mathcal{F}_T$ from $\{P_2, P_3\}$
16) $\neg$Conc $\rightarrow \neg P_3$ is provable in $\mathcal{F}_T$ from $\{P_1, P_2\}$
17) $(P_1 \wedge P_2 \wedge P_3) \rightarrow$ Conc is provable in $\mathcal{F}_T$ from $\{\ \}$
18) $(\neg P_1 \wedge \neg P_2 \wedge \neg P_3) \rightarrow \neg$Conc is not provable in $\mathcal{F}_T$ from $\{\ \}$
19) $P_1 \rightarrow (P_2 \rightarrow (P_3 \rightarrow \text{Conc}))$ is a logical truth
20) $\neg\text{Conc} \rightarrow (\neg P_1 \wedge \neg P_2 \wedge \neg P_3)$ is a logical truth

Part III. Which of the above MUST be false?